CYBERNETICS
and COMPUTER
TECHNOLOGIES

UDC 519.85 DOI:10.34229/2707-451X.20.1.0

B. GOLDENGORIN, V. KUZMENKO, MK-S TSO

PRICING BY THE ALLOCATION MODEL WITH DIFFERENT TYPE OF DISCOUNTS

Abstract. *In this paper we apply the Capacitated Facility Location Problem with two new types of discounts to find a best allocation of customer's demands on the given set of potential suppliers. Its distinctive features are that instead of usual production costs and unique capacity level of supplier we consider customer costs and several possible sizes for the capacity level which are measured by costs (available amounts of money), respectively. On the set of multiple capacities of money we define customer cost functions with two new types of discounts which lead to a discontinuous and neither concave nor convex function. Such cost functions have practical value in handling cases in which multiple capacities can be represented by a collection of embedded money amounts such that on the maximum values of capacities we may apply both types of discounts. We present results of computational experiments based on two different implementations of the branch-and-bound type algorithms which show that application of scale economies discounts reduces the total computation times for finding an optimal solution.*

Keywords: *allocation, decision support systems, integer programming, location.*

Introduction. We consider a problem motivated by the following common situation. A company acting as an e-trading agent takes enquiries from a number of potential customers. Each enquiry requests a price quotation for a basket of goods consisting of specified quantities of certain products which may be automobile components, pharmaceuticals, resources on a shared telecommunications facility etc. The enquiry will be converted to a firm order if the unit price of the individual products and the total price of the basket of goods are judged to be acceptable. The price quotation will be based on the best prices available to the company from a set of suppliers in the wider market. These prices may or may not be available to the customers of the e-trading company. However the advantage to potential customers of dealing with the e-trading entity may derive from a number of factors.

1) The e-trading company may provide a convenient "one stop shop" supported by e-banking arrangements for its customers, for whom it becomes uneconomic to consult the numerous alternative suppliers in the market.

2) The company may be able to negotiate better "static" prices than its customers by its greater purchasing power.

3) The company may be able to achieve a "dynamic" economy of scale effect by opportunistic aggregation of orders.

A key requirement for the company is to find the best purchase price from the wider market for a specific basket of goods. We will assume that the same goods may be obtained from different suppliers at different prices, though no single supplier has uniformly the best price. We will also assume a fixed cost associated with the use of any supplier, e.g. a charge for carriage and order administration, which may be waived (discounted) on larger orders exceeding a certain value. Further, we refer to this type of discounts as type A. It is common also to obtain a fixed percentage discount from the price list on orders above a certain value threshold, namely type B, which is also known as scale economies. Possibly more than one discount threshold applies as well as both types of discounts simultaneously. There is therefore an incentive to use to fewer suppliers to take advantage of these two types of discount. ……………

Model formulation. Allocation models very often reflect the relationships between fixed investments and current costs (profits) for different type of economic activities. Every decision to invest in a new activity leads to a specific price list of products and services in both private and public sectors reflecting the trade off between a profit of potential producers and saved payment of customers. This paper introduces a new modification of the CFLP with two different types of discounts for customer (production) costs per unit of product. In essence, this model can be considered as a tool for analyzing the existing set of price lists with purpose to find a ‘best’ hybrid of two different types of discounts. Its distinguishing feature is that output from any given facility is measured in monetary amount with prohibited intervals such that the incremental output can be achieved at either additional or reduced costs. Thus, it introduces a range of capacity constraints a violation each of which can be done with more attractive for producers and customers list of prices by applying of two types of discounts.

The first type of discounts is studied by a number of researches in the framework of the Uncapacitated Facility Location Problem (UFLP), and was modeled by concave non-decreasing production cost function (see e.g. Feldman et al., 1966; Efromson and Ray, 1966; Soland, 1974; Van Roy and Erlenkotter, 1982; Krarup and Pruzan, 1983). This type of production cost function reflects the so called economies of scale effect, in which the unit production costs are declining with increased scale of output. The technique for formulating the concave production cost in the framework of UFLP is realized by introducing a set of pseudo-facilities each of which is associated with a piecewise linear segment, as illustrated in Fig. 1 for three pseudo-facilities.

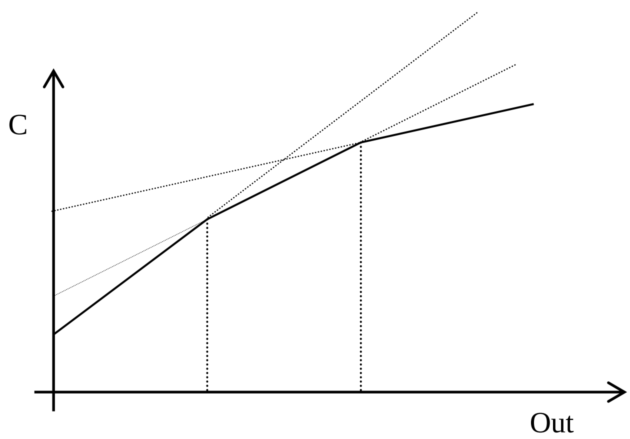

FIG. 1. Concave production function with discounts of type B

The original optimization problem has been formulated as follows:

$$\min_{y_i, x_{ij}} \left\{ \sum_{i=1}^{n} f_i y_i + \sum_{i=1}^{n} \sum_{j=1}^{m} c_{ij} x_{ij} \right\}, \quad (1)$$

subject to

$$L_i y_i \le \sum_{j=1}^{m} c_{ij} x_{ij} \le U_i y_i, \quad i = 1,...,n, \tag{2}$$

$$\sum_{i_1 \in I_i^d} y_{i_1} \le 1, \quad i \in I_0, \tag{3}$$

$$\sum_{i=1}^{n} x_{ij} \ge 1, \quad j = 1,...,m, \tag{4}$$

$$y_i \in \{0, 1\}, \quad 0 \le x_{ij} \le 1 \text{ or } x_{ij} \in \{0, 1\}. \tag{5}$$

The model (1)-(5) is the so called SPLP with lower and upper bounds $L_i y_i \le \sum_{j=1}^{m} c_{ij} x_{ij} \le U_i y_i, \quad i = 1,...,n$, and additional constraints $\sum_{i_1 \in I_i^d} y_{i_1} \le 1, \quad i \in I_0$ reflecting a discount procedure which is depending on the total sum of the so called transportation costs. It means that values of f_i, c_{ij}, L_i, U_i will be pre-specified for each specific instance of (1)–(5).

In the model (1)–(5) the demands are scaled like in an usual model of the Simple Plant Location Problem (SPLP). All entries of the input file - f_i, c_{ij}, L_i, U_i must be computed according the specific requirements involved in the model (1)–(5). Here I_0 is the set of original suppliers, I_i^d - the set of pseudo-suppliers, corresponding the initial supplier i and its range of discounts for the pre-specified costs functions. Sets I_i^d are pair-wise disjoint, i.e. for any $i_1, i_2 \in I_0$ holds $I_{i_1}^d \cap I_{i_2}^d = \varnothing$.

The collection of all I_i^d is equal to the set of pseudo-suppliers $\bigcup_{i \in I_0} I_i^d = I$ with $I = \{1,...,n\}$; y_i is a Boolean variable reflecting the use of service from pseudo-supplier .

Solving of the dual problem. The method of solving the problem (6)-(10) is based on the solution tree in each node of which the dual problem (11)-(17) is solved. Depending on the pegged (permanently fixed) and fixed (temporary fixed) values of Boolean variables y_i, each node of the solution tree by sets K_0, K_1, K_2 is defined. The values of P_L, P_U are uniquely defined by the previous node of the solution tree in the parent level after solving the corresponding dual problem. The first dual problem will be solved on the root of the solution tree which is defined by $P_L = 0$, $P_U = n$, $K_0 = \varnothing$, $K_1 = \varnothing$, $K_2 = I$.

We have studied and tested two algorithms for solving the dual problem. The first algorithm is due to Beasley, and the second algorithm is the r-algorithm which is based on the space dilation in the direction of difference of two successive subgradients (see Shor 1998). …………………

Branching. Very often after solving the first dual problem most of the variables y_i are remaining as not fixed. The fixing of such variables is determined by usual b-n-b method. The number of subproblems created during the execution of b-n-b method depends on the chosen variable as well as its value: either 0 or 1. Which variable to chose, and which value to assign to the chosen variable constitute the branching rule. We consider three stages for our branching rule. …………………..

Numerical Example. First of all we have considered a small illustration example.

We are given 3 suppliers each of which sales 5 types of flowers.

The suppliers costs and discounts functions are defined as follows:

1-st supplier. The basic fixed costs is 3$.
 1-st discount: from 50$ the basic fixed costs is 0$, and discount is 3%.
 No other discounts.
2-nd supplier. The basic fixed costs is 3$.
 1-st discount: from 50$ the basic fixed costs is 0$, and discount is 0%.
 No other discounts.
3-d supplier. The basic fixed costs is 3$.
 1-st discount: from 50$ the basic fixed costs is 0$, and discount is 2%.
 2-nd discount: from 100$ the basic fixed costs is 0$, and discount is 4%.
 No other discounts.

The basic demands in flowers and their prices is shown in Table 1.

TABLE 1. The basic demands in flowers and their prices

Flower Name	Demands, items	Basic price, $	Order Values, $
Aster	10	2,40	24,00
Bamboo	10	1,60	16,00
Lily	20	1,00	20,00
Orchid	12	3,00	36,00
Violet	30	0,80	24,00
	Overall order value		120,00

Results. ……………

Conclusions. We have addressed a general problem of importance in e-commerce, how to determine a minimum cost assignment of an order for a basket of goods to a set of suppliers taking into account fixed charges and some common types of discounting policies. The fast response time required in an online context motivates the need for an efficient computational procedure. ………..……

Authorship contribution statement.
Boris Goldengorin: investigation, conceptualization, methodology, formal analysis, writing – original draft.
Viktor Kuzmenko: software, visualization.
Mike Tso: supervision, formal analysis, resources, writing – review & editing.

Data availability statement. The data that supports the findings of this study is openly available in ‘‘Mendeley data’’ at https://data.mendeley.com/datasets/nrkx767tf/.

Acknowledgment. If any. Grants. Support. Data.

Appendix. If any.

(References in citation order)

If original reference is written by Cyrillic alphabet then it should be duplicated by Roman alphabet in such manner:

authors' names should be transliterated;
papers' or books' titles should translated into English;
journals' or publishers' names should be transliterated;
for other output data only number should be used

Original language should be specified (in brackets) if it is not English.
Hyperlink should be specified too (if any). And first of all the DOI.
CrossRef service https://apps.crossref.org/SimpleTextQuery can be use to find DOIs.
Searching results can depend on style of citation. So, results should be verified.

More commonly used styles are here https://libguides.williams.edu/citing/acs

Citation in References is closer to ACS-style

References

1. Metrological support. (in Russian) http://metrob.ru/html/mo/ (accessed: 01.02.2020)
2. Internet image collection ImageNet. http://www.image-net.org (accessed: 13.01.2020)
3. Vdovychenko R.O. Implementation of Distributed Memory for modern GPU and investigation of features of the model. *Kompyuterna matematyka*. 2019. **1**. P. 77–84. (in Ukrainian) http://dspace.nbuv.gov.ua/handle/123456789/161936
4. Beasley J.E. Lagrangean heuristics for location problems. *European Journal of Operational Research*. 1993. **65**. P. 383–399. https://doi.org/10.1016/0377-2217(93)90118-7
5. Mikhalevich V.S., Trubin V.A., Shor N.Z. *Optimization problems for production and transportation planning.* M.: Nauka, 1986. 264 p. (in Russian)
6. Kochetov Yu., Ivanenko D. Computationally Difficult Instances for the Uncapacitated Facility Location Problem *Proceedings of the 5th Metaheuristic Conference (MIC 2003)*. Kyoto. 2003.

Received 13.04.2018.

Boris Goldengorin,
professor, head of department, University of Groningen, The Netherlands,
https://orcid.org/0000-0001-7399-581X
goldengorin@gmail.com

Viktor Kuzmenko,
PhD, senior researcher, V.M. Glushkov Institute of Cybernetics of the NAS of Ukraine, Kyiv,
https://orcid.org/0000-0001-7284-3662

Mike Tso,
PhD, lecturer, University of Manchester, Manchester, UK.
https://orcid.org/0000-0002-8276-202X

Two extended annotations in Ukrainian and in English. They are needed for website. Every extended annotations should contain not less that 1800 characters with keywords (near 2/3 of page with title and keywords). Extended annotations will also be printed.

If the authors cannot prepare extended annotation in Ukrainian then English extended annotation will be translated.

MSC 90B85

Boris Goldengorin [1 *], Viktor Kuzmenko [2], Mike Tso [3]

Pricing by the Allocation Model with Different Type of Discounts

[1] *University of Groningen, The Netherlands*
[2] *V.M. Glushkov Institute of Cybernetics of the NAS of Ukraine, Kyiv*
[3] *University of Manchester, Manchester, UK*
[*] *Correspondence: goldengorin@gmail.com*

Introduction. We consider a problem motivated by the following common situation. A company acting as an e-trading agent takes enquiries from a number of potential customers. Each enquiry requests a price quotation for a basket of goods consisting of specified quantities of certain products which may be automobile components, pharmaceuticals, resources on a shared telecommunications facility etc. The enquiry will be converted to a firm order if the unit price of the individual products and the total price of the basket of goods are judged to be acceptable. The price quotation will be based on the best prices available to the company from a set of suppliers in the wider market. These prices may or may not be available to the customers of the e-trading company.

The purpose of the paper. This paper introduces a new modification of the CFLP with two different types of discounts for customer (production) costs per unit of product. In essence, this model can be considered as a tool for analyzing the existing set of price lists with purpose to find a 'best' hybrid of two different types of discounts. Its distinguishing feature is that output from any given facility is measured in monetary amount with prohibited intervals such that the incremental output can be achieved at either additional or reduced costs.

Results. The method of Bilde and Krarup is easy to implement and has low space requirements, but exhibits slow convergence for non-linear production functions, especially with a ravine-like objective function. The r-algorithm is relatively difficult to implement and needs more computation time, but solves very well hard instances which give rise to a ravine-like objective function.

Conclusions. We conclude that Lagrangean relaxation techniques can efficiently solve to optimality the problem of allocating a basket of goods to a set of suppliers at minimum cost, when two types of discount (percentage and reduction in fixed cost) are applied. Future work will be aimed at using these optimal solutions as a tool for evaluating alternative discount strategies. It would be very interesting to examine how such a tool may help to create price lists with a flexible discount structure reflecting the current competition in a specific market for products and services.

Keywords: allocation, decision support systems, integer programming, location.

УДК 519.85

Б. Голденгорін [1 *], В. Кузьменко [2], М. Тсо [3]

Ціноутворення за допомогою моделі розміщення з різними видами знижок

[1] *Університет Гронінгену, Нідерланди*
[2] *Інститут кібернетики імені В.М. Глушкова НАН України, Київ*
[3] *Університет Манчестера, Великобританія*
[*] *Листування: goldengorin@gmail.com*

Вступ. Ми вважаємо проблему мотивованою наступною загальною ситуацією. Компанія, яка виступає агентом електронної торгівлі, приймає запити у ряду потенційних клієнтів. Кожен запит повторно шукає котирування цін на кошик товарів, що складається з визначеної кількості певної продукції, наприклад, автомобільні компоненти, фармацевтичні препарати, ресурси спільного телекомунікаційного об'єкту тощо. Окремі товари та загальна ціна кошика товарів вважаються прийнятними. Котирування цін буде базуватися на найкращих цінах, доступних компанії від набору постачальників на більш широкому ринку. Ці ціни можуть бути або не бути доступними для клієнтів компанії з електронної торгівлі.

Мета роботи. У цій статті представлена нова модифікація CFLP з двома різними видами знижок для замовника на одиницю товару. По суті, ця модель може розглядатися як інструмент для аналізу наявного набору прейскурантів з метою пошуку «найкращого» гібриду двох різних типів знижок. Її особливістю є те, що вихід з будь-якого об'єкта вимірюється у грошовій сумі в певному інтервалі, таким чином, щоб додатковий обсяг виробництва був досягнутий або за додаткові, або за зменшені витрати.

Результати. Дослідження показали, що метод Більде та Крарупа легко реалізувати і він має низькі вимоги до пам'яті, але демонструє повільну збіжність для нелінійних виробничих функцій, особливо при ароподібній цільовій функції. Натомість, r-алгоритм важче реалізувати і він потребує більше часу на обчислення, але він розв'язує складні задачі, у яких цільова функція є подібною до яру.

Ключові слова: розподілення, системи підтримки прийняття рішень, цілочисельне програмування, розміщення.

A Ukrainian copy of References. If a list of references contains items translated from Ukrainian, Russian or other languages then these items must be written in original language. Such list will be used on Ukrainian pages of website.

Список літератури

1. Метрологическое обеспечение. http://metrob.ru/html/mo/ (звернення: 01.02.2020)
2. Internet image collection ImageNet. http://www.image-net.org (звернення: 13.01.2020)
3. Вдовиченко Р.О. Реалізація розподіленої пам'яті на сучасних графічних процесорах і дослідження характеристик моделі. *Комп'ютерна математика*. 2019. **1**. С. 77–84. http://dspace.nbuv.gov.ua/handle/123456789/161936
4. Beasley J.E. Lagrangean heuristics for location problems. *European Journal of Operational Research*. 1993. **65**. P. 383–399. https://doi.org/10.1016/0377-2217(93)90118-7
5. Михалевич В.С., Трубин В.А., Шор Н.З. *Оптимизационные задачи производственно-транспортного типа*. М.: Наука, 1986. 264 с.
6. Kochetov Yu., Ivanenko D. Computationally Difficult Instances for the Uncapacitated Facility Location Problem *Proceedings of the 5th Metaheuristic Conference (MIC 2003).* Kyoto. 2003.