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FAST INTEGER SINE-COSINE TRANSFORMS OF ORDER 4 AND SIMPLIFIED SINE-COSINE TRANSFORMS OF ORDER 8

A matrix method for constructing one-norm sine-cosine transforms of type II of order 4 has been developed, which has better efficiency than the known sine transform of type II. An integer one-norm sine-cosine transform of type II of order 4 has been proposed, and on its basis an integer one-norm simplified sine-cosine transform of order 8 of low computational complexity has been developed. Fast algorithms for the proposed transforms have been considered. The computational complexity of the simplified sine-cosine transform of type II of order 8 is only 40 operations, which is three times less than the computational complexity of the known sine transform of type VII of order 8, and the compression ratio is 1.5–2.3 % lower. The proposed integer one-norm simplified sine-cosine transform of order 8 can be used for image and signal analysis and coding tasks, in particular for separable adaptive transforms as an alternative to the sine transform of type VII for high-speed coding modes.

Keywords: *discrete sine-cosine transform, integer sine-cosine transform, simplified sine-cosine transform, adaptive separable transforms, factorization.*

Introduction. Discrete cosine and sine transforms of various types and their integer approximations have been successfully studied for a long time, including the development of fast calculation algorithms [1–17]. Kekre and Solanki [6] introduced the discrete sine-cosine transform of type II, which, according to their estimates, is the closest to DCT-II compared to other transforms. In [10], it was shown that the sequential application of 1D DCT and chipboard transforms is an alternative to the 2D DCT-II transform, and a mode-dependent directional transform was introduced, which adaptively chooses one of the four possible combinations of DCT/DST. Among the sine transforms, the type VII sine transform is considered the most effective, in particular, the H.265 standard uses an integer one-norm type VII sine transform of order 4 [18], and the VVC standard [19, 20] uses higher orders. In [15], an optimal fast algorithm for calculating the generalized integer one-norm sine transform of type VII of order 4, which requires 5 multiplications and 11 additions, was proposed . As an alternative to the sine transform of type VII of order 8, a modified sine-cosine transform of type VII of order 8 was proposed in [16], obtained on the basis of the sine transform of type VII of order 4 and the cosine transform of type II of order 4. In this work, other alternatives to the sine transform of type VII of order 8 are proposed: a simplified sine-cosine transform of type II of order 8, obtained on the basis of two sine-cosine transforms of type II of order 4, and a sine transform of type II of order 8.

Modification of the type II sine transform of order 4 to obtain the type II sine-cosine transform of order 4

The purpose of the modification of the known sine transform of type II of order 4 is to increase the efficiency of the transform, in particular, as an alternative to the computationally more complex sine transform of type VII in separable adaptive transforms. The modification scheme for the integer sine transform of order 4 includes the use of two values of the parameters of the transforms instead of three, as well as the replacement of two rows of the sine transform with similar rows of the cosine transform of order 4, which as a result forms a sine-cosine transform of order 4. In other words, the scaled Hadamard matrix of the order 2,which is a component of the type II sine transform of order 4, is replaced by the matrix of the type II cosine transform of order 2, which is a component of the cosine transform of order 4. A similar substitution can be applied in sine transforms of type II of any order, but as the order increases, the efficiency of the added cosine transform of order 2 instead of the Hadamard transform of order 2 will decrease proportionally. The computational complexity as a result of such a replacement can both increase and decrease, depending on the specific values of the parameters of the transforms, in particular, for matrices with the norm of the number of the power of two – increase.

Integer transforms of order 4 of low complexity

Integer transforms of various types with fast calculation algorithms are widely used in various fields. The simplest of these is the Hadamard transform, which has a fast, low-complexity algorithm with no multiplication operations, and has one default norm. Due to the low computational complexity, the Hadamard transform has numerous applications, despite its lower data compression efficiency (about 3–5 % less than the cosine transform for order 4, 6–10 % less for order 8, and 10–15 % less for order 16, depending on the correlation of the input data). Haar, Haar-Hadamar transforms also have linear computational complexity, but require additional computing resources for normalization operations.

On the other hand, there are one-norm integer transforms, which have lower computational complexity than their counterparts with different norms. For example, in the H.264 codec, an integer cosine transform of order 4 is used, which requires only 12 operations, if you do not take into account normalization, for which the normalization matrix is used, which increases the computational complexity by 12 multiplication operations.

The tasks of the Internet of Things require efficient transforms of low computational complexity, on the basis of which simplified codecs are built [21].

The method of constructing a sine-cosine transform of order 4

The pattern of cosine transform of type II of order 4 has the form:

$$C_4 = \begin{bmatrix} k & k & k & k \\ i & j & -j & -i \\ k & -k & -k & k \\ j & -i & i & -j \end{bmatrix}, \ i > j . \tag{1}$$

Similarly, the sine transform of order 4 will have the form

$$S_4 = \begin{bmatrix} j & i & i & j \\ k & k & -k & -k \\ i & -j & -j & i \\ k & -k & k & -k \end{bmatrix}. \tag{2}$$

If we take the lines with the parameters ***i*** and ***j*** from the cosine transform and replace them with the lines with the parameter ***k*** of the sine transform, we get a sine-cosine transform that has only two parameters ***i*** and ***j*** and takes the following form:

$$SC_4 = \begin{bmatrix} j & i & i & j \\ i & j & -j & -i \\ i & -j & -j & i \\ j & -i & i & -j \end{bmatrix}, \ i > j . \tag{3}$$

It is easy to consider the scheme of construction by using a matrix of the order 2×2 cosine transform instead of the matrix of the scaled Hadamard transform, which is a component of the sine transform of type II of order 4.

The sine transform of order 4 consists of two transforms of order 2:

$$S_2 = \begin{bmatrix} j & i \\ i & -j \end{bmatrix}, \quad T_2 = \begin{bmatrix} k & k \\ -k & k \end{bmatrix}, \tag{4}$$

because

$$S_4 = diag\left[S_2, T_2\right] H_4,$$
$$H_4 = \begin{vmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 1 & 0 & 0 & -1 \end{vmatrix}. \tag{5}$$

As it is easy to see, the transform T_2 is scalable with the parameter ***k*** the well-known 2×2 Hadamard transform:

$$H_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}. \tag{6}$$

As you know, the Hadamard transform is worse in efficiency than the sine and cosine transforms, so it seems reasonable to replace it with a more complex transform of order 2, for example, taken from the cosine transform of order 4.

The cosine transform of order 4 also consists of two transforms of order 2

$$C_4 = diag\left[T_2, C_2\right] H_4,$$
$$T_2 = \begin{bmatrix} k & k \\ k & -k \end{bmatrix}, C_2 = \begin{bmatrix} j & i \\ -i & j \end{bmatrix}. \tag{7}$$

Accordingly, we can take the matrix C_2 from the cosine transform, and get a new transform consisting of a sine and a cosine transform of order 2:

$$SC_4 = diag\left[S_2, C_2\right] H_4 . \tag{8}$$

Thus, the obtained sine-cosine transform of type II of order 4 should theoretically have better efficiency than the traditional sine transform of type II of order 4.

Note that the cosine transform of order 4 also includes as a component the scaled 2×2 Hadamard transform, and it could be similarly replaced by a 2×2 sine transform with parameters ***i*** and ***j***, but the result would end up to be the same sine-cosine transform.

For the development of a new sine-cosine transform with one norm, it is necessary to select three parameter values (k, i, j) according to the requirement of one norm

$$2k^2 = i^2 + j^2 . \tag{9}$$

For order 4, this problem is quite simple, but very few possible sets of values fully satisfy this condition, such as the well-known transform ($k = 13, i = 17, j = 7$), which is completely orthogonal and has no normalization error. Taking into account the small error of deviation from one norm, there are more transform options, for example, ($k=7, i=9, j=4$), which has a normalization error of 1 % for the 2 D option and 0.5 % for the scheme of separate adaptive transforms.

For parameters ($k=7, i=9, j=4$), the matrix of an integer sine-cosine transform of type II of order 4 will have the following form:

$$SC_4^{(7)} = \begin{bmatrix} 4 & 9 & 9 & 4 \\ 9 & 4 & -4 & -9 \\ 9 & -4 & -4 & 9 \\ 4 & -9 & 9 & -4 \end{bmatrix}. \quad (10)$$

And for ($k=13, i=17, j=7$), respectively

$$SC_4^{(13)} = \begin{bmatrix} 7 & 17 & 17 & 7 \\ 17 & 7 & -7 & -17 \\ 17 & -7 & -7 & 17 \\ 7 & -17 & 17 & -7 \end{bmatrix}. \quad (11)$$

In terms of computational complexity, the transform $SC_4^{(7)}$ will have an advantage because it requires 4 multiplications, 8 additions, and 4 shifts, i.e., 16 operations, or 20 additions and shifts if you replace each multiplication by 9 with 2 shifts and additions. In addition, the normalization operation will also have low computational complexity in the scheme of separable adaptive transforms. The second transform $SC_4^{(13)}$ will have slightly better accuracy for small quantization parameters QPstepst 1 to 8, but higher computational complexity: 8 multiplications and 8 additions, or 24 addition and shift operations with no multiplication operations, and higher computational complexity in normalization.

In [15], a fast algorithm for calculating the sine transform of type VII of order 4, which has 5 multiplication operations and 11 addition operations, was proposed. Thus, it is impractical to use the proposed sine-cosine transform instead of the type VII sine transform of order 4, alternative transforms of lower computational complexity are more necessary for order 8, where there are no fast algorithms for the type VII sine transform .

A fast algorithm for calculating the sine - cosine transform of type II of order 4

Matrix core SC_4 sine-cosine transform of order 4 can be represented as a product of two matrices:

$$SC_4 = diag\left[S_2, C_2\right] H_4, \quad (12)$$

where matrices S_2, C_2, H_4 have the form:

$$H_4 = \begin{vmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 1 & 0 & 0 & -1 \end{vmatrix}, \quad S_2 = \begin{bmatrix} j & i \\ i & -j \end{bmatrix}, C_2 = \begin{bmatrix} j & i \\ -i & j \end{bmatrix}. \quad (13)$$

Simplified sine-cosine transform of order 8

On the basis of two sine-cosine transforms of order 4, a sine-cosine transform of order 8 can be constructed, which will have less computational complexity and, accordingly, less efficiency than a standard sine or cosine transform of type II of order 8, so it can be called a simplified sine-cosine transform.

This method is well-known, for example, with its use a transform of order 8 was constructed on the basis of two transforms of order 4, namely, a sine transform of type VII and a cosine transform of type II in the work [16], and can be represented as

$$T_8 = diag\left[T_4^{(1)}, T_4^{(2)}\right] H_8 . \tag{14}$$

In our case $T_4^{(1)} = T_4^{(2)} = SC_4$, that is

$$SC_8 = diag\left[SC_4, SC_4\right] H_8 , \tag{15}$$

where $H_8^* = \begin{bmatrix} I_4 & \bar{I}_4 \\ \bar{I}_4 & -I_4 \end{bmatrix}$, $\bar{I}_4 = antidiag\left[I_4\right]$. I_4, $\bar{I}_4$ are unit diagonal and antidiagonal 4×4 matrices.

Since the first row of the matrix is sine, this determines that the transform can be used in the scheme of discrete adaptive transforms instead of the sine transform, despite the presence of cosine functions. Similarly, hybrid cosine-sine transforms can be applied instead of the cosine transform for discrete transforms.

The simplified sine-cosine transform matrix for the parameters (k, i, j) will have the following generalized form:

$$SC_8 = \begin{bmatrix} j & i & i & j & j & i & i & j \\ j & i & i & j & -j & -i & -i & -j \\ i & j & -j & -i & i & j & -j & -i \\ i & j & -j & -i & -i & -j & j & i \\ i & -j & -j & i & i & -j & -j & i \\ i & -j & -j & i & -i & j & j & -i \\ j & -i & i & -j & j & -i & i & -j \\ j & -i & i & -j & -j & i & -i & j \end{bmatrix} . \tag{16}$$

Let's remember that the parameter ***k*** determines the norm of the matrix, and is defined as

$$2k^2 = i^2 + j^2 . \tag{17}$$

To normalize separate transforms, the obtained transform coefficients are known to be divided by $k_1 k_2 N$, where ***N*** is the order of transform.

The matrix of the simplified sine-cosine transform for parameters ($k = 7, i = 9, j = 4$) will have the following form:

$$SC_8 = \begin{bmatrix} 4 & 9 & 9 & 4 & 4 & 9 & 9 & 4 \\ 4 & 9 & 9 & 4 & -4 & -9 & -9 & -4 \\ 9 & 4 & -4 & -9 & -9 & -4 & 4 & 9 \\ 9 & 4 & -4 & -9 & 9 & 4 & -4 & -9 \\ 9 & -4 & -4 & 9 & 9 & -4 & -4 & 9 \\ 9 & -4 & -4 & 9 & -9 & 4 & 4 & -9 \\ 4 & -9 & 9 & -4 & -4 & 9 & -9 & 4 \\ 4 & -9 & 9 & -4 & 4 & -9 & 9 & -4 \end{bmatrix} . \tag{18}$$

The use of two identical lower-order transforms can be considered a partial case of the method of obtaining hybrid transforms based on lower-order transforms. For example, it is possible to propose a simplified cosine transform of order 8 of reduced complexity relative to the traditional cosine transform of order 8 based on the type II cosine transform of order 4.

A fast algorithm for calculating a simplified integer sine-cosine transform of type II of order 8

The matrix of CS_8 the sine-cosine transform of order 8 can be factorized as the product of three matrices:

$$CS_8 = T_8^{(3)} T_8^{(2)} T_8^{(1)}, \tag{19}$$

where $T_8^{(k)}$ are k-th, $k=\overline{1,3}$, 8×8 factor matrix of the proposed fast algorithm for calculating the simplified sine-cosine transform

$$T_8^{(1)} = H_8,\ T_8^{(2)} = diag\left[H_4, H_4\right],\ T_8^{(3)} = diag\left[S_2, C_2, S_2, C_2\right]. \tag{20}$$

The matrices S_2, C_2 have the form:

$$S_2 = \begin{bmatrix} j & i \\ i & -j \end{bmatrix}, C_2 = \begin{bmatrix} j & i \\ -i & j \end{bmatrix}. \tag{21}$$

The calculation scheme of the simplified sine-cosine transform of order 8 will have 2 times more operations than the sine-cosine transform of order 4, and additionally 8 operations for the matrix H_8, that is, 48 operations in the case without using multiplication operations, or 32 operations of addition and shift and 8 multiplication operations, for parameters ($k=7,\ i=9,\ j=4$), which together makes 40 operations.

Simplified modes of separable adaptive transforms

The computational complexity when using separable adaptive transforms increases due to the need to sort and compare 4 possible combinations of sine and cosine transforms. Accordingly, to reduce computational costs, two additional simplified modes are proposed: mode A involves the use of two combinations out of four, namely 2D cosine and cosine/sine combination, and the mode B uses 3 combinations, without sine/sine transform. Mode B has considerably better compression ratio than mode A so it is of more interest value. Additionally, in case of the presented simplified sine-cosine transform, the sine/sine combination is the least likely, so mode B is optimal for it.

Integer sine and cosine transforms of order II for separable adaptive transforms

For a simplified quantization scheme that uses values of the quantization parameter that are equal to the power of two, it is possible to consider such pairs of separable adaptive transforms that, with compatible quantization, will have minimal computational complexity. For example, if we use the cosine transform of order 4 from [22] with $\boldsymbol{k}$ = 18, and the sine transform with $\boldsymbol{k}$ = 7, then the quantization operation will include division by $7\cdot 18\cdot 4$, that is, division by 63 and 8, which is replaced by multiplication by 65 (which can be replaced by two operations - addition and shift) and the corresponding shift, which is combined into one shift with $2\cdot 4$.

Experimental results

The results of comparative testing of the simplified sine-cosine transform of order 8 versus well-known sine transform of type VII and II on test images are presented. The program uses an adaptive choice of five value prediction algorithms, namely DC / V / H / diag45 ° / diag135 °, and simplified R – D modeling (dead zone quantifier for AC, RLE with a zero counter, probabilistic bit estimate through cross entropy by RUN / | LEVEL | / EOB), with a choice of zigzag from three options. Several images of different types were tested, primarily with a low correlation level of pixel values: landscape (Fig. 1), grass [23], and similar compression estimates were obtained for two typical quantization parameters QPstep = 18 and QPstep = 28: 2 % and 2.3 % lower compression than sine type VII for landscape, 1.7 % and 2.3 % – for grass for the full mode of separable adaptive transforms.

FIG. 1. Landscape test image

Similar results were also obtained for the simplified three-combination mode (excluding the sine/sine combination): 1.6 % and 2 % lower compression than the VII type sine for the landscape image, and 1.8 % and 2 % for the grass image.

Thus, on average, the proposed simplified transform provides 2 % less compression coefficient, and at the same time provides an additional 1 % to 3 % increase in compression compared to only 2D cosine.

A modified II type sine transform (with modified having a consistent advantage of 0.3 % higher compression ratio than the classical) was also tested and showed higher results than the proposed simplified transform, namely: 0.6 % and 1 % higher compression ratio for the full separable adaptive transform mode and 0.5 % and 0.6 % higher compression for the simplified three-combination mode, with minimal deviations for different image types.

Thus, the II type sine can also be used in the separable adaptive transform scheme as an alternative to the sine type VII for high-speed modes, because despite having 1.2–1.4 % less compression coefficient it has fast calculation algorithm with 46 total operations, 14 of which are multiplications (same computational complexity as cosine transform of type II).

Considering that the computational complexity of the sine transform of type VII of order 8 is 120 operations, i.e. 3 times more than that of the developed simplified sine-cosine transform of type II, it is possible to adaptively apply the simplified transform for fast modes [21] for low correlation and for high compression ratios.

Let's consider the testing results of the proposed simplified sine-cosine transform of order 8 on the models, and calculate the characteristics of the coding gain and the efficiency of the transform on the isotropic model.

The results of testing according to the characteristics of the efficiency of the transform are shown in Fig. 2.

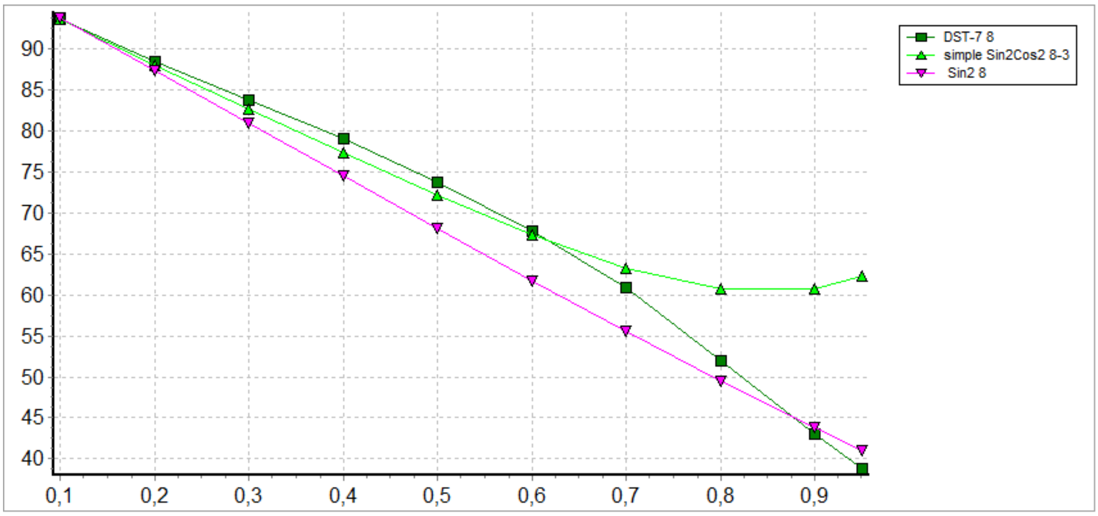

FIG. 2. The results of testing according to the transform efficiency characteristic

The results of coding efficiency gain according to characteristic 1 D G_{TC} is given in Table.

TABLE. Results of comparison by the coding efficiency for different correlation coefficients for 1D isotropic Markov process

Transform for blocks 8×8	Evaluation of coding efficiency according to characteristic 1D G_{TC} (dB) for correlation coefficients $\rho_{\upsilon}, \rho_h = 0,2 \div 0,7$ for 1D isotropic Markov process					
	0.2	0.3	0.4	0.5	0.6	0.7
DCT- 8	0.14	0.34	0.63	1.05	1.64	2.50
IST-VII -8	0.15	0.34	0.63	1.04	1.60	2.38
Simple Sin-Cos-II -8	0.14	0.32	0.58	0.94	1.43	2.10
IST-II - 8	0.14	0.31	0.56	0.89	1.32	1.89

According to the results of tests on models, the proposed simplified sine-cosine transform of order 8 has better performance than sine transform of type II of order 8, and according to the characteristics the efficiency of the transform is quite close to sine transform of type VII of order 8.

Conclusions. A method for constructing integer sine-cosine transforms of type II of order 4 was developed. An integer one-norm sine-cosine transform of type II of order 4 was developed. A simplified integer sine-cosine transform of type II of order 8 was proposed based on the developed sine-cosine transform of order 4. Fast calculation algorithms were considered for the proposed transforms. The computational complexity of the simplified sine-cosine transform of type II of order 8 is only 40 operations, which is three times less than the computational complexity of the known sine transform of type VII of order 8, and the compression ratio is 1.5–2.3 % lower. Simplified modes for separable adaptive transforms are proposed, for a fast codec it is suggested to use 2–3 combinations instead of 4 possible. As an alternative to the type VII sine transform in the separable transform scheme for adaptive application in high-speed modes, a modified type II sine transform is proposed, which consistently compresses 0.3 % better than the classic type II sine transform, and lags behind the type VII sine transform by 1.2–1.4 %.

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Швидкі цілочислові синус-косинусні перетворення порядку 4 і спрощені синус-косинусні перетворення порядку 8

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Вступ. Розроблено матричний метод побудови однонормових синус-косинусних перетворень типу II порядку 4, який має кращу ефективність порівняно з відомим синусним перетворенням типу II. Запропоновано цілочисельне однонормове синус-косинусне перетворення типу II порядку 4, а на його основі розроблено цілочисельне однонормове спрощене синус-косинусне перетворення порядку 8 із низькою обчислювальною складністю. Розглянуто швидкі алгоритми обчислення для запропонованих перетворень. Обчислювальна складність спрощеного синус-косинусного перетворення типу II порядку 8 становить лише 40 операцій, що втричі менше за обчислювальну складність відомого синусного перетворення типу VII порядку 8, а коефіцієнт стиснення на 1,5–2,3 % нижчий, як показують представлені експериментальні результати. Запропоноване цілочисельне однонормове спрощене синус-косинусне перетворення 8-го порядку може бути використане для задач аналізу та кодування зображень та сигналів, зокрема для роздільних адаптивних перетворень як альтернатива синусному перетворенню VII типу для високошвидкісних та екстремальних режимів кодування. Запропоновано два спрощені режими для адаптивних роздільних перетворень: режим A, який використовує дві комбінації з чотирьох загальних, а саме 2D косинус та косинус/синус; режим B, який використовує три комбінації з чотирьох, виключаючи варіацію синус/синус. Як альтернатива синусного перетворення типу VII у схемі роздільних перетворень для адаптивного застосування в швидкодійних режимах пропонується модифіковане синусне перетворення типу II, яке стабільно на 0,3 % краще стискає за класичне синусне перетворення типу II, і на 1,2–1,4 % відстає від синусного перетворення типу VII.

Ключові слова: дискретне синус-косинусне перетворення, цілочисельне синус-косинусне перетворення, спрощене синус-косинусне перетворення, адаптивні роздільні перетворення, факторизація, обчислювальна складність, перетворення Адамара, перетворення Хаара, перетворення Хаара – Адамара, гібридні перетворення.